Solutions of final exam questions

Set A = B = C

Q1. True / false

(i) T (ii) T (iii) T (iv) F (v) T (vi) T

(vii) T (viii) T (ix) F (x) T

Q3. For steady, two-dimensional flow, the rate of change of temperature is

$$\frac{dT}{dt} = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = (x^2 - y^2 + x)\,8x + (-2xy - y)(-9y^2)$$

At $(x, y) = (2, 1)$, $\quad \frac{dT}{dt} = (5)(16) - 5(-9) = 125$ units **(Ans)**

Q3 Incompressible Continuity equation is given by

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

$$\Rightarrow \frac{\partial}{\partial x}(4xy^2) + \frac{\partial}{\partial y}(-2y^2) = 4y^2 + \frac{df}{dy} - y^2 = 0$$

$$\Rightarrow \frac{df}{dy} = -3y^2, \quad \text{Integrate: } f(y) = \int (-3y^2)\,dy = -y^3 + \text{constant}$$

(Ans)

Q4 The relevant dimensions are $\tau_w = \{ML^{-1}T^{-2}\}$, $\rho = \{ML^{-3}\}$, $\mu = \{ML^{-1}T^{-1}\}$
$\Omega = \{T^{-1}\}$, $R = \{L\}$ and $\Delta r = \{L\}$, with $n = 6$, $m = 3$, we expect
$n - m = 3$ pi groups. They are found, as specified using $[\rho, \Omega, R]$ as repeating variables

$$\Pi_1 = \rho^a \Omega^b R^c \tau_w = \left[\frac{M}{L^3}\right]^a \left[\frac{1}{T}\right]^b [L]^c \left[\frac{M}{LT^2}\right] = M^0L^0T^0, \text{ solve: } a=-1, b=-2, c=-2$$

$$\Pi_2 = \rho^a \Omega^b R^c \mu^{-1} = \left[\frac{M}{L^3}\right]^a \left[\frac{1}{T}\right]^b [L]^c \left[\frac{M}{LT}\right]^{-1} = M^0L^0T^0, \text{ solve: } a=1, b=1, c=2$$

$$\Pi_3 = \rho^a \Omega^b R^c \Delta r = \left[\frac{M}{L^3}\right]^a \left[\frac{1}{T}\right]^b [L]^c [L] = M^0L^0T^0, \text{ solve } a=0, b=0, c=-1$$

The final dimensionless function has the form:

$$\Pi_1 = f(\Pi_2, \Pi_3) \quad \text{or,} \quad \frac{\tau_{wall}}{\rho \Omega^2 R^2} = f\left(\frac{\rho \Omega R^2}{\mu}, \frac{\Delta r}{R}\right) \quad \text{(Ans)}$$

Q5 (a) For steady flow, we have

$$Q_1 + Q_2 + Q_3 = Q_4 \quad \text{or} \quad V_1A_1 + V_2A_2 + V_3A_3 = V_4A_4 \quad \text{— (1)}$$

Since $0.2\,Q_3 = 0.1\,Q_4$ & $Q_4 = 120\,(m^3/h)\cdot\frac{h}{3600\,s} = 0.0333\,m^3/s$

$$V_3 = \frac{Q_4}{2A_3} = \frac{0.0333\,m^3/s}{\frac{\pi}{2}(0.06)^2} = 5.89\,m/s \quad \text{(Ans(b))}$$

Substituting into (1)

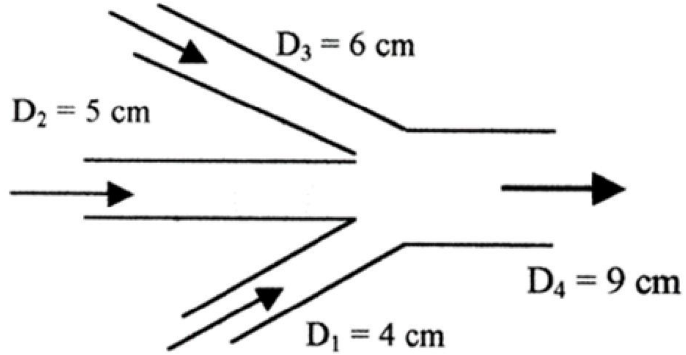
$$V_1\left(\frac{\pi}{4}\right)(0.04)^2 + 5\left(\frac{\pi}{4}\right)(0.05)^2 + 5.89\left(\frac{\pi}{4}\right)(0.06)^2 = 0.0333$$

$\Rightarrow V_1 = 5.45\,m/s$ (Ans (a))

From mass conservation, $Q_4 = V_4A_4$

$$(0.0333\,m^3/s)^* = V_4(\pi)(0.06)^2/4$$

$\Rightarrow V_4 = 5.24\,m/s$ Ans (c)



Q6. The CV (control volume) should surround the tank and wheel and cut through the cable and the exit water jet. Then the horizontal force balance is

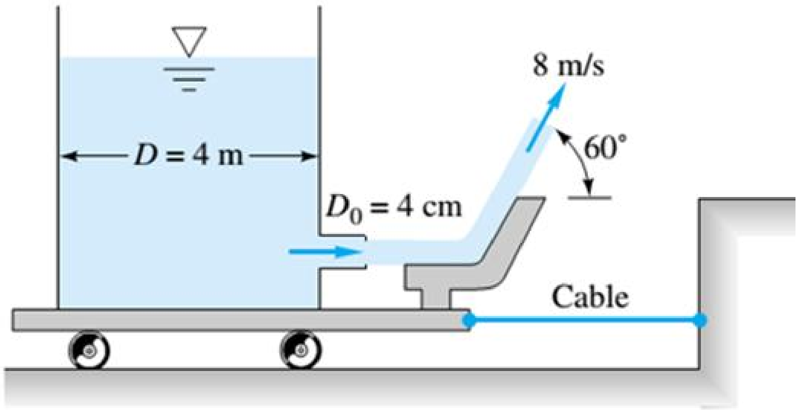
$$\sum F_x = T_{cable} = \dot{m}_{out} V_{out} = (\rho A V_j) V_j \cos\theta$$

$$= 1000 \left(\frac{\pi}{4}\right) (0.04)^2 \, 8^2 \cos 60^\circ \simeq 40 \text{ N}$$

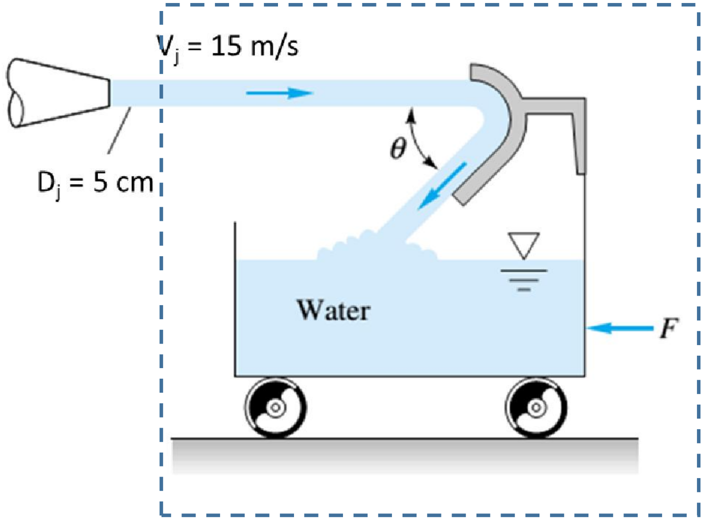
(Ans)

THINK

What will happen when $\theta = 0^\circ$



Q7. The control volume surrounds the tank and wheels and cuts through the jet as shown.



We have to assume that splashing into the tank does not increase the x-momentum of the water in the tank. Then we can write the CV horizontal force relation

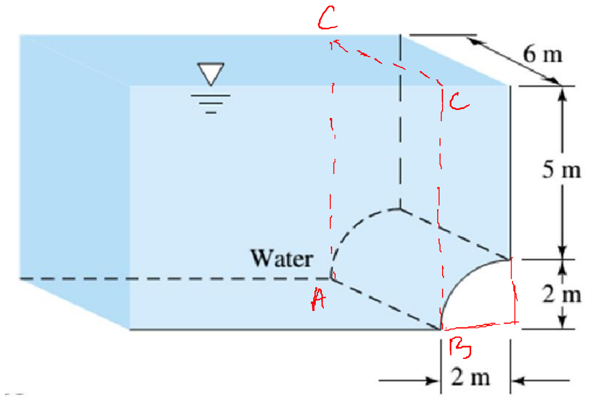
$$\sum F_x = -F = \frac{d}{dt}\left(\int u \rho \, dV\right)_{tank} = -\dot{m}_{in} u_{in} = 0 - \dot{m} V_{jet}$$

(independent of θ)

Thus

$$F = (\rho A_j V_j) V_j = 1000 \times \frac{\pi}{4} \cdot \left(\frac{5}{100}\right)^2 \cdot 15^2 \cong 441 \text{ N. (Ans)}$$

Q8. The horizontal component is



$$F_H = \rho g h_{cg} A_{vert.}$$

$$= 1000 \times 9.8 \times (5+1) \times (2 \times 6)$$

$$\simeq 705600 \text{ N} \qquad \underline{\text{Ans (a)}}$$

The vertical component is the weight of fluid above the quarter-circle panel

F_V = weight of parallelopiped − weight of quarter circle

$$= mg_{Box} - mg_{1/4 circle} = 1000 \times (2 \times 7 \times 6) \times 9.8 - 1000 \times 9.8 \times \frac{\pi}{4} \cdot 2^2 \times 6$$

$$\simeq 638000 \text{ N}$$

<u>Ans (b)</u>

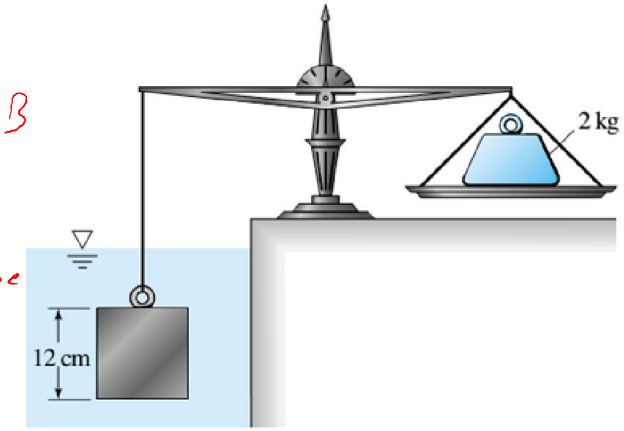
Q9.

The scale force is $2 \times 9.81 = 19.62$ N.

The specific weight of ethanol is 7733 N/m^3

Then, $F = 19.62 = (\text{Weight} - \text{Buoyancy force})_{cube}$

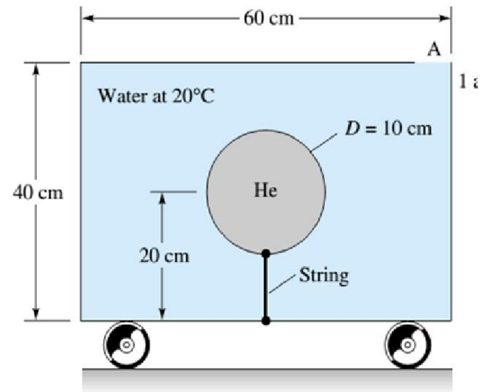
$= (\gamma_{cube} - 7733)(0.12)^3$



Solve for γ_{cube}: $\gamma_{cube} = 7733 + \dfrac{19.62}{0.12^3} \approx 19100$ N/m^3

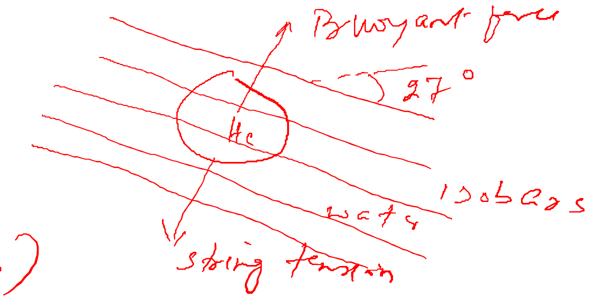
Ans.

Q10 The acceleration sets up pressure isobars which slant down and to the right, in both water and helium. This means there will be a buoyancy force on the balloon up and to the right as shown.



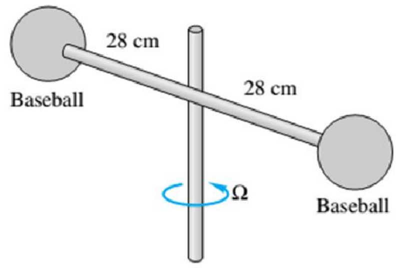
It must be balanced by the string tension down and to the left. If we neglect the balloon material weight, the balloon leans up & to the right at angle

$$\theta = \tan^{-1}\left(\frac{a_x}{g}\right) = \tan^{-1}\left(\frac{5}{9.81}\right) \approx 27^\circ \text{ (Ans)}$$



measured from the vertical. This acceleration-buoyancy effect may seem counter-intuitive

Q11. $\rho = 1.225\ kg/m^3$, $\mu = 1.78e\text{-}5\ kg/m\text{-}s$, $C_D = 0.47$ (ball), $C_D = 1.2$ (rod)



power = Force × velocity

In this case, $P = 2 F_b V_b + 2 F_{rod} V_{rod}$

$\omega = 400\ rpm \times 2\pi/60 = 41.9\ rad/s$

Each ball moves at a centerline velocity, $V_b = \omega r_b = 41.9(0.28 + 0.0735/2)$
$= 13.3\ m/s$

Drag force on each ball, $F_b = C_D\left(\frac{1}{2} A \rho V^2\right)_{ball} = 0.47\left(\frac{1.225}{2}\right)(13.3)^2 \cdot \frac{\pi}{4} \times (0.0735)^2$
$= 0.215\ N$

Rod

$V_r = \omega r_{avg} = 41.9 \times 0.14 = 5.86\ m/s$

$F_{rod} = C_D \times \left(\frac{1}{2} A \rho V^2\right)_{rod} = 1.2\left(\frac{1.225}{2}\right)(5.86)^2 \times (0.007)(0.28) = 0.0495\ N$

$P = 2 F_b V_b + 2 F_r V_r \approx 6.3\ W$ (Ans)