Dynamics of Double-Diffusive Finger Convection: Structures and Convective Fluxes

A Thesis

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By

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Abstract

The main focus of this thesis has been to understand the dynamics of double-diffusive finger convection and investigate the role of various parameters that govern the finger structures, velocity, fluxes and their interdependence. Double-diffusive convection occurs in the presence of two components with different molecular diffusivity. When the faster diffusing component (T) stabilizes the system ($d\rho/dz < 0$) and the slower diffusing component (S) makes the system unstable ($d\rho/dz > 0$), with overall stratification remaining gravitationally stable ($d\rho/dz < 0$), the convection takes the form of alternatively rising and sinking cells called 'salt fingers'. ρ is the total density of the fluid. In this thesis, results from the numerical simulations, analytical and laboratory investigations of the dynamics of finger convection has been presented for a wide ranges of governing parameters.

Fingers can form when $1 \leq R_\rho \leq \tau^{-1}$. Here, τ is the diffusivity ratio of the slower to the faster diffusing component and R_ρ is the ratio of density contributions of the faster to the slower diffusing component. Previous idealized linear models for a linearly stratified double-diffusive system predicts finger width with infinite wavelength when the R_ρ approaches either 1 or τ^{-1}. However, in laboratory experiments, fingers width of finite size have always been observed when R_ρ is either close to one or near marginal state. We solved the equations for growth rate analytically for the fastest growing normal mode and found that the finger width remains finite even at $R_\rho = 1$. We have shown that the inconsistency in the earlier theoretical predictions is on the account of *ad hoc* assumptions made in their analysis.

In this thesis, we have solved numerically the partial differential equations governing the continuity of mass, momentum, energy and species in two-dimension. We have considered a two-layer system similar to the laboratory setup. A series of simulations have been conducted in the heat-salt system at a fixed R_ρ close to one ($= 1.001$) for Rayleigh numbers ranging from 7×10^3 to 7×10^8. We have identified many interesting features which has not been reported previously: (a) at high thermal Rayleigh number (Ra_T), where thin fingers evolve, the salinity (S) and temperature (T) fields are not distinguishable from each other, (b) at low Ra_T, wide fingers evolve, and the effect of molecular diffusion is observed in the T fields only. As the system moves from being advection dominated at high Ra_T to diffusion dominated at low Ra_T, velocity in the fingers decreases substantially.

In this thesis, we have demonstrated that Rayleigh number plays a significant role in determining the initial instability that develops at the interface and controls the dynamics of finger convection. Simulations were carried out for R_ρ ranging from 1.001 to 10 and for the ranges of Ra_T from 10^4 to 10^9 . The density stability ratio is shown to be less important in controlling the instability compared to Rayleigh number. This result is in contrast to the previous investigations that has given significant weight to the only non-dimensional parameter, namely

R_ρ. We find that the convection onset time, t_0 varies as $t_0 \sim Ra_S^{-3/5}$. Before and after the onset of instability, we observed the two important features: (a) $\left|d\overline{S}/dt\right| < \left|d\overline{T}/dt\right|$, before the onset and, (b) $\left|d\overline{S}/dt\right| > \left|d\overline{T}/dt\right|$, after the onset. Here $\overline{S}$ and $\overline{T}$ are the time averaged temperature and salinity in each layer. These conditions are satisfied in each layer for all values of R_ρ and Ra_T in the fingering regime covered in the present studies.

In the range of the values of R_ρ and Ra_T used in the simulations, we found that the effect of Ra_T is more pronounced on finger structures than R_ρ. The non-dimensional width of the fingers (δ_f/H), in a two layer system, varies as $Ra_T^{-1/3}$, where δ_f is the finger width and H is the domain height. This relation is justified by scale analysis. Linear theory predicts the variation as $(\delta_f/H) \sim Ra_T^{-1/4}$ when the stratification is linear. With the increase in finger width at low Rayleigh numbers, velocity in the fingers decreases. For a given value of Ra_T and R_ρ, the maximum value of kinetic energy attained by the system varies as $Ra_T^{2/3} R_\rho^{-3/4}$. Velocity scale and finger width are inversely related. In the range of parameters used, we found the relationship as $(velocity\ scale) \sim (finger\ width)^{-0.85}$. This relationship is justified with a simple scale analysis.

At high Ra_T, the system passes through five phases of evolution while at low Ra_T, only first two phases are observed. Due to the dominance of thermal diffusion in the first phase at low Ra_T, the instantaneous value R_ρ can decrease even below one. This results in the overturning convection. However, at high Ra_T, mixed layer forms above and below the finger zone. Strong convection in the adjoining layers does not allow the fingers to grow further. Overturning convection was not observed at high Ra_T. We conducted laboratory experiments at high and low Rayleigh numbers and we observed features similar to those obtained in the simulations.

Flux ratio, R_f, is the ratio of convective flux of heat to salt. The amount of heat and salt transported vertically is strongly dependent on the finger velocity and width. Rayleigh number and density ratio controls the finger width and velocity. We have shown that the effect of Rayleigh number is more important than R_ρ. Almost all the previous studies have investigated the effect on fluxes as a function of R_ρ only. We found that the value of R_f is centered around 0.6 at high $Ra_T = 7 \times 10^8$ for $R_\rho = 1.5$, 2, 6, and 10. The value of R_f at $Ra_T = 7 \times 10^3$ for the above values of R_ρ is less then 0.2. Clearly, R_f is a strong function of Rayleigh numbers and a weak function of R_ρ, which was not realized in the previous studies. We also found that the statement, "when $R_\rho \to 1, R_f \to 1$" is correct only at high Rayleigh numbers. Another interesting result is that wide salt fingers have low flux ratio. This is not predicted by the linear analysis. Idealized linear theories predicts high flux ratio for wider fingers compared to the thin fingers.

We have examined the effect of diffusivity ratio ($\tau = k_S/k_T$), Prandtl number ($Pr = \nu/k_T$) and gravity (g) on salt finger structures. Diffusivity of the faster diffusing component (k_T) was kept constant while the diffusivity of the slower diffusing component (k_S) was varied such that $\tau \to 1$. Rayleigh number was varied from 7×10^3 to 7×10^8 at high value of τ. We conclude that the effect of Ra_T and R_ρ on finger flux ratio is significant only when τ of the system is low. At higher value of τ, the structure of S and T fields are similar for *any* value of Ra_T and R_ρ. A similar

structure for both the fields results in a high value of R_f. Effect of gravity on finger structures was studied. Fingers were allowed to evolve initially at low g and then suddenly subjected to full gravity. We observed that the initial condition of the system plays a crucial role in determining the finger structures in the new environment.

Effect of Prandtl number on the finger system has been studied both numerically and with the help of laboratory experiments. Increase in Pr of the system was achieved by increasing the viscosity of the fluid (ν). Evolution of the salt finger structures were studied numerically for three cases: $Pr = 7$, $Pr = 7 \times 50$ and $Pr = 7 \times 400$. Simulating higher Pr system was computationally expensive. We observed interesting transition of finger system from low Pr to high Pr. At fixed R_ρ, thin fingers evolve initially and later, strong convecting regions developed above and below the finger zone in the low Pr system. This limits the growth of the fingers. As Pr increases, wide fingers evolve and the sandwich structures observed at low Pr slowly disappears. This allows the fingers to penetrate deep into the reservoirs. Salt fingers in high Pr system never reach equilibrium. In laboratory experiments, Pr of the system was increased by adding carboxy-methyl cellulose (CMC) to distilled water. CMC increases the viscosity of the fluid exponentially without affecting the density of the fluid. Experiments were performed in salt-sugar and heat-sugar systems. We observed features that were similar to the numerical simulations at high and low Pr. Observations from laboratory and numerical simulations were extrapolated to suggest a new model for the formation of columnar basalt structures.